

# **Radiant Shock Waves in the Post-Quasistatic Approximation**

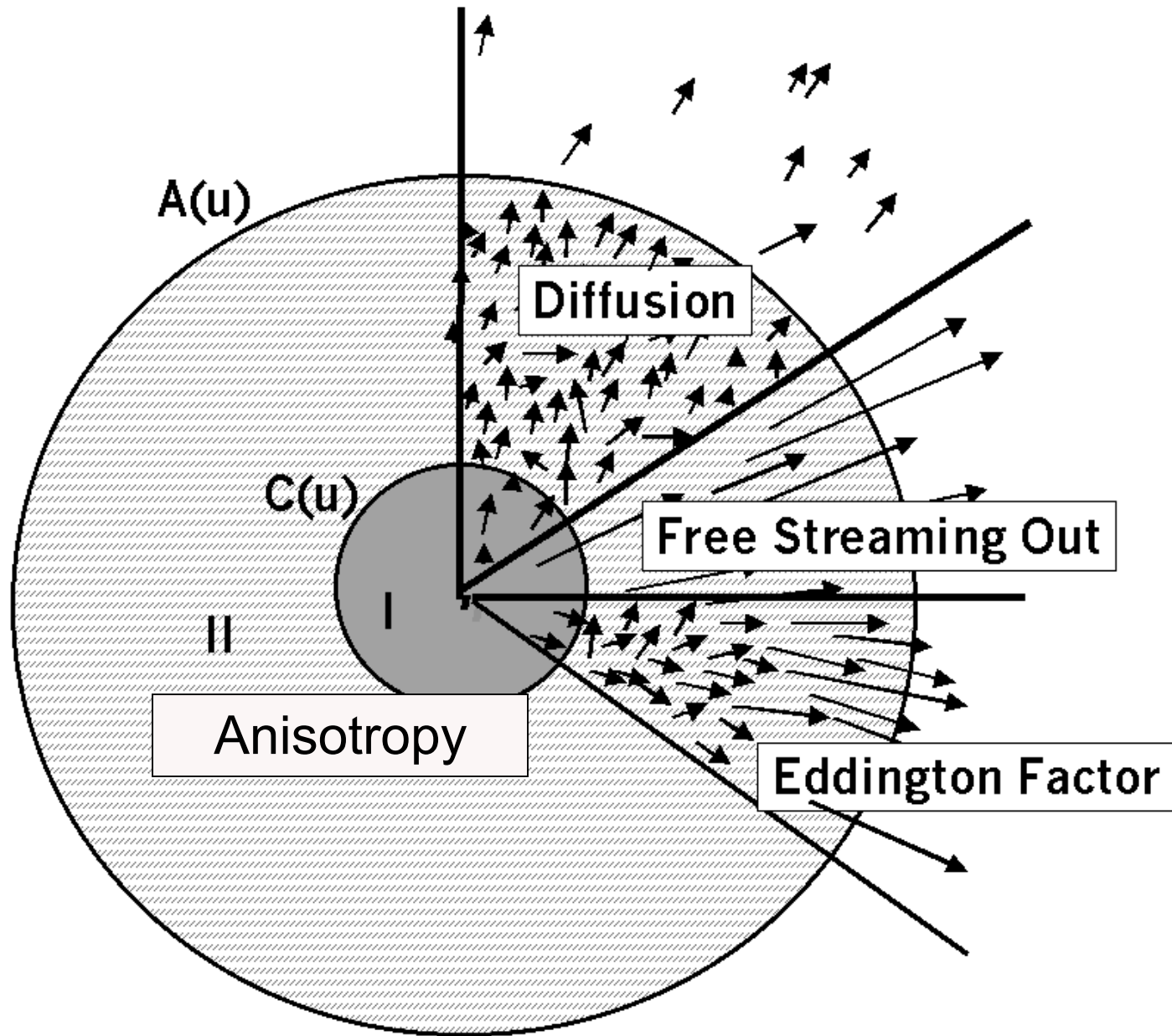
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# The physical problem



# Semi Numeric Methods

## Analytical + Numeric



**Einstein  
Equations**

**Mild Physical  
Assumptions**



**Ordinary  
Differential Equations  
at the Surfaces**

# Matter Configurations & Radiation Processes

Isotropic  $P_{\perp} = P_r$

Anisotropic

Charged

Viscous

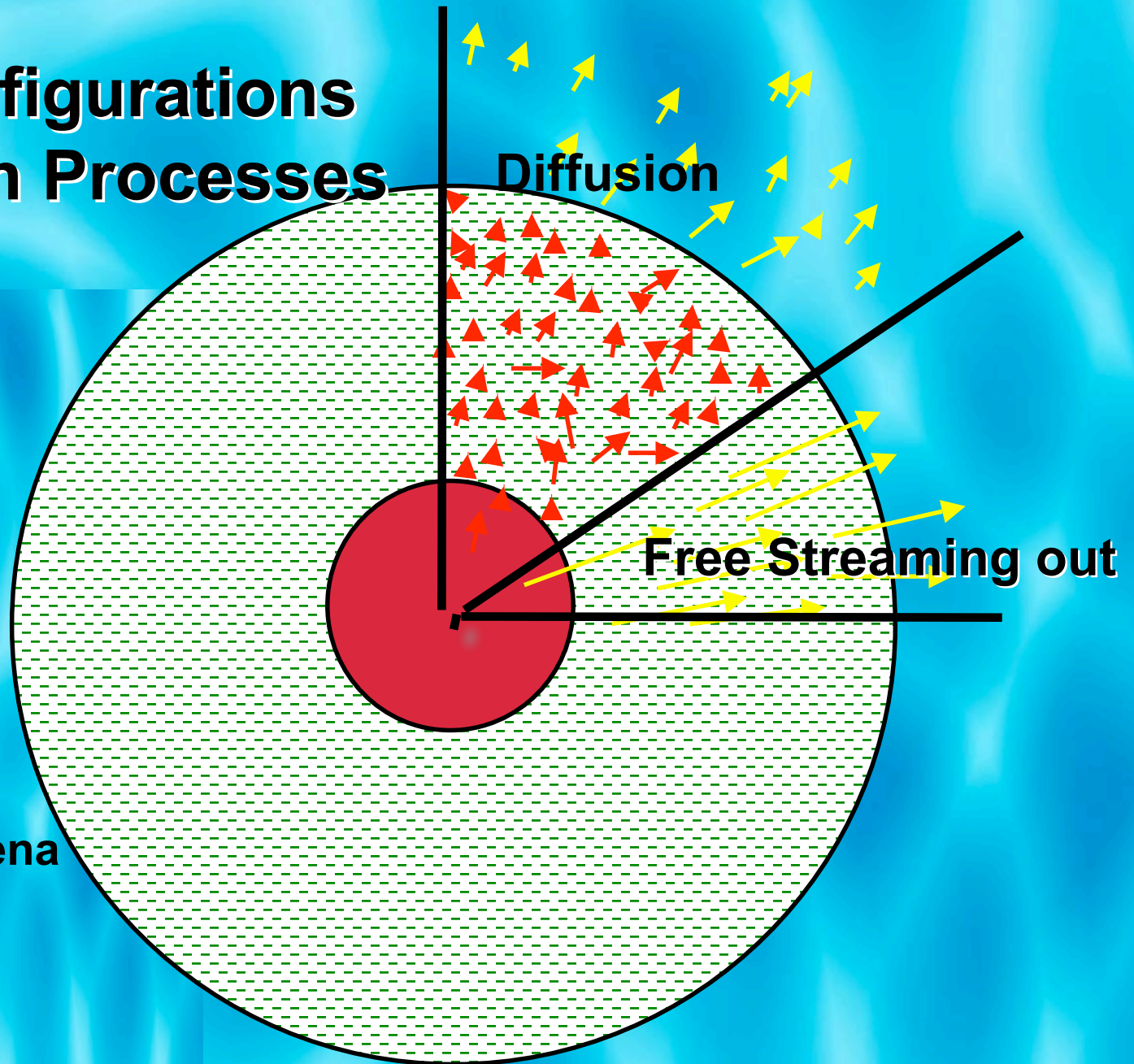
Shocks

Oscillations

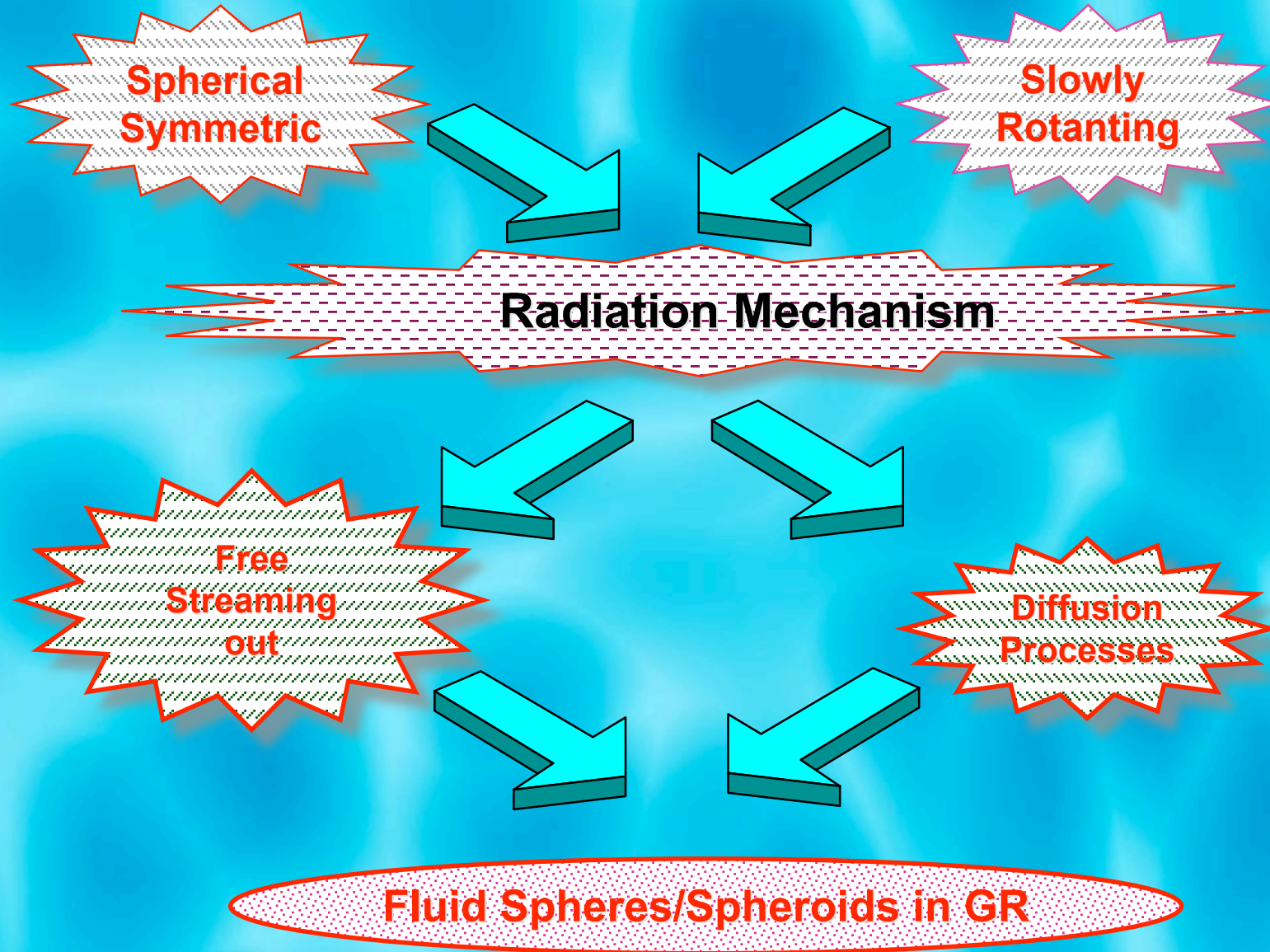
Homologous

Slow Rotation

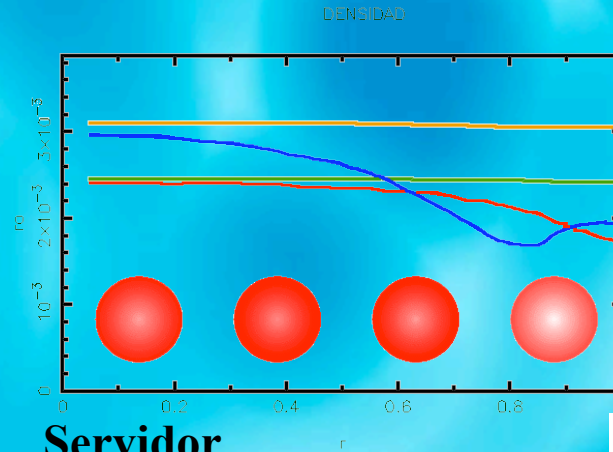
Surface Phenomena



# Mild Physical Assumptions



# The GR collapse physics on the WEB



Cliente



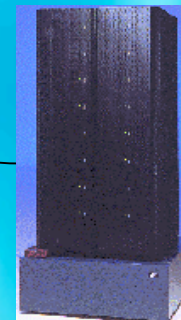
Servidor  
WEB



Interacción con  
el usuario



Cálculo Serial



Cálculo  
Paralelo

# The GR collapse physics on the WEB

**CeCalCULA**

**ESFERAS**

$G_{\mu\nu} = 8\pi T_{\mu\nu}$

**Condiciones Iniciales**

$t_0$   $t_f$   $t$  # de capas

0.000000 30.000000 0.100000 20

$\rho_0$   $\omega$

05.404 1.000000

$\eta$

0.000000

masa radiada pico  $\sigma$

0.0010000 10.00000 1.000000

carga anisotropia

0.000000 1.000000

RUN

**Animación de la salida de Esferas**

En esta animación se muestran las cuatro variables físicas en su evolución temporal.

Para activar la animación escoja la velocidad haciendo clic en alguno de los botones y luego presione el botón "animar". También puede ir a un cuadro en particular de la animación escogiendo en el cuadro de la derecha y presionando en el botón "refrescar"

Para ver una de las variables en más detalle puede hacer click sobre el círculo que la representa

Velocidad de la animación

- Rápido
- Medio
- Lento
- Más Lento

Animar

Ir al cuadro (1 - 20):

8 refrescar

passo = 36

**Animación de la salida de Esferas**

En esta animación se muestran la evolución temporal de la Velocidad Radial.

Para activar la animación escoja la velocidad haciendo clic en alguno de los botones y luego presione el botón "animar". También puede ir a un cuadro en particular de la animación escogiendo en el cuadro de la derecha y presionando en el botón "refrescar"

Para volver a la página anterior puede hacer click en el botón de retorno de su navegador o puede hacer click sobre la imagen

Velocidad de la animación

- Rápido
- Medio
- Lento
- Más Lento

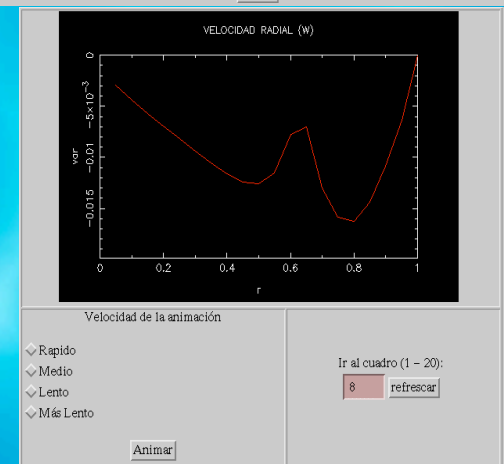
Animar

Ir al cuadro (1 - 20):

8 refrescar

Para ver un gráfico XY de la evolución de esta variable

ventana



**CeCalCULA**

**ESFERAS**

$G_{\mu\nu} = 8\pi T_{\mu\nu}$

**Configuración Básica**

Modelo: Schwarchild

Métrica: neutra

Tensor de Energía-momentum: Anisótropo, esc. libre, neutro

Prueba: no

Su dirección de correo: gutierrez@cecalc.ula.ve

Su dirección de correo será utilizada sólo para identificar la ejecución de los procesos de cada usuario, en ningún momento será incluida en listas de correos ni será cedida a terceros.

Seguir

# Loong time ago...

We have studied this problems in Bondi Radiation coord.

$$ds^2 = e^{2\beta} \left\{ \frac{V}{r} du^2 + 2dudr \right\} - r^2 d\theta^2 - \sin^2 \theta r^2 d\phi^2$$

Herrera + Núñez (1990). **Evolution of Radiating Spheres in General Relativity: A Seminumerical Approach.** *Fundam. of Cosmic Physics*, 14, 235-319 (1990).

**Today... a more intuitive vision...**

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

With the most richer environment we can simulate

$$T_{\mu\nu} = \left( \rho + P + \frac{1}{2}(\rho_R - \mathcal{P}) \right) u_\mu u_\nu - \left( P + \frac{1}{2}(\rho_R - \mathcal{P}) \right) g_{\mu\nu} \\ + \left( P + \mathcal{P} - \left( P + \frac{1}{2}(\rho_R - \mathcal{P}) \right) \right) \chi_\mu \chi_\nu + \mathcal{F}_{(\mu} u_{\nu)}$$

# Radiation Hydrodynamics scenario

$$d\mathcal{E} = \mathbf{I}(r, t; \vec{n}, \nu) dS \cos \varphi d\Theta d\nu dt,$$

$$\rho_R = \frac{1}{2} \int_0^\infty d\nu \int_1^{-1} d\mu \mathbf{I}(r, t; \vec{n}, \nu), \quad \mathcal{F} = \frac{1}{2} \int_0^\infty d\nu \int_1^{-1} d\mu \mu \mathbf{I}(r, t; \vec{n}, \nu)$$

$$\mathcal{P} = \frac{1}{2} \int_0^\infty d\nu \int_1^{-1} d\mu \mu^2 \mathbf{I}(r, t; \vec{n}, \nu)$$

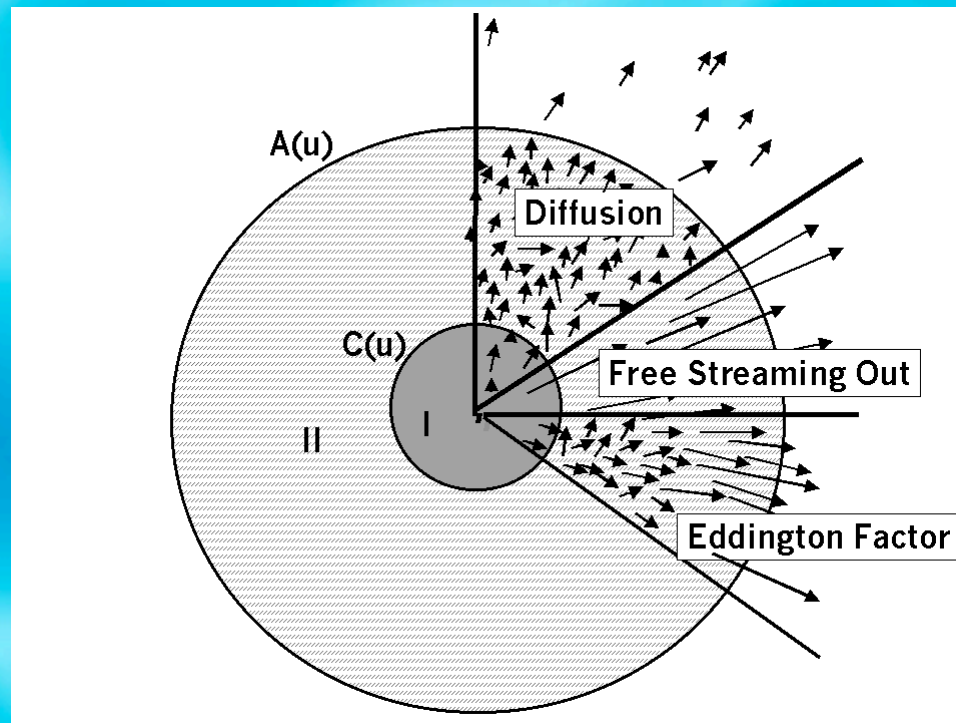
with  $f = \frac{\mathcal{F}}{\rho_R}$  and  $\chi = \frac{\mathcal{P}}{\rho_R}$

$$\left. \begin{array}{l} \mathcal{F} = \mathcal{P} = \rho_R \Rightarrow f = 1 \quad \chi = 1 \\ \mathcal{P} = \frac{1}{3} \rho_R \Rightarrow f \rightarrow 0; \quad \chi = \frac{1}{3} \end{array} \right\} \Rightarrow 0 \leq f \leq 1 \quad \text{and} \quad \frac{1}{3} \leq \chi(f) \leq 1$$

$$\|f\| \leq 1, \quad f^2 \leq \chi \leq 1 \quad \text{and} \quad -\frac{1-\chi}{1+f} \leq \frac{d\chi}{df} \leq \frac{1-\chi}{1-f}$$

# Junction Conditions

- On the shock
  - First + Second Fundamental Forms
  - Rankine Hugoniot Conditions  $[T_{\alpha\beta}n^\beta]_c = 0$
- On the boundary: First + Second Fundamental Forms



# Now the effective variables

$$\tilde{P} = \frac{(P_r + P_R) + (\rho + \rho_R)\omega^2 + 2\omega\mathcal{F}}{1 - \omega^2}$$

$$\tilde{\rho} = \frac{(\rho + \rho_R) + (P_r + P_R)\omega^2 + 2\omega\mathcal{F}}{1 - \omega^2}$$

Instead of

$$\tilde{P} = \frac{P + \mathcal{P} - \mathcal{F} - \omega_x(\rho + \rho_R - \mathcal{F})}{1 + \omega_x} \equiv \frac{P - \omega_x\rho + \frac{1}{f}(\chi - f - \omega_x(1 - f))\mathcal{F}}{1 + \omega_x}$$

$$\tilde{\rho} = \frac{\rho + \rho_R - \mathcal{F} - \omega_x(P + \mathcal{P} - \mathcal{F})}{1 + \omega_x} \equiv \frac{\rho - \omega_x P + \frac{1}{f}(1 - f - \omega_x(\chi - f))\mathcal{F}}{1 + \omega_x}$$

And that is why this is post quasistatic

# Seminumeric Approach

$$\tilde{\rho}(u, r) = \rho_{st}(r)\rho_d(u) \quad \tilde{P}(u, r) = P_{st}(r)P_d(u)$$

$$\rho_{st}(r = a(u))\rho_d(u) = \tilde{\rho}(A, F, \Omega, \text{and derivatives})$$

$$P_{st}(r = a(u))P_d(u) = \tilde{P}(A, F, \Omega, \text{and derivatives})$$

## Field equations at the surface

$$G_{\mu\nu} \left( \beta \left( \tilde{\rho}(u, r), \tilde{P}(u, r) \right), \tilde{m} \left( \tilde{\rho}(u, r), \tilde{P}(u, r) \right), \text{and derivatives} \right) = T_{\mu\nu} (\rho, \rho_R, P, \mathcal{P}, \mathcal{F}, \omega_r, \omega_\phi, \mathcal{D})$$

## Surface equations

$$\dot{A} = F(\Omega - 1)$$

$$\frac{\dot{F}}{F} = \frac{2L + (1 - F)(\Omega - 1)}{A}$$

$$0 = \frac{\dot{\Omega}}{\Omega} + \frac{\dot{F}}{F} + \frac{(\tilde{\rho}_a)_{,0}}{\tilde{\rho}_a} + \frac{F\Omega^2 \tilde{R}}{\tilde{\rho}_a} - \frac{2F\Omega}{A\tilde{\rho}_a} \left( P_a + \frac{\chi(f)\mathcal{F}_a}{2f} (\chi(f) - 1) \right) + (\Omega - 1) \left( \frac{F\Omega\tilde{\rho}_{1a}}{\tilde{\rho}_a} - \frac{4\pi A(1 - 3\Omega)\tilde{\rho}_a}{\Omega} - \frac{3 + F}{2A} \right)$$

## Metric elements

$$\beta(u, r) = \int_a^r 2\pi\bar{r} \frac{\tilde{\rho} + \tilde{P}}{\left(1 - \frac{2\tilde{m}}{\bar{r}}\right)} d\bar{r} \quad \text{and} \quad \tilde{m}(u, r) = \int_0^r 4\pi\bar{r}^2 \tilde{\rho} d\bar{r}$$

## Physical Variables

$$\text{with } f = \frac{\mathcal{F}}{\rho_R} \quad \text{and} \quad \chi = \frac{\mathcal{P}}{\rho_R}$$

$$G_{\mu\nu} \left( \beta \left( \tilde{\rho}(u, r), \tilde{P}(u, r) \right), \tilde{m} \left( \tilde{\rho}(u, r), \tilde{P}(u, r) \right), \text{and derivatives} \right) = T_{\mu\nu} (\rho, \rho_R, P, \mathcal{P}, \mathcal{F}, \omega_r, \omega_\phi, \mathcal{D})$$

# The modeling: effective variables

**Nucleus: Schwarzschild--like model**

$$\tilde{\rho}_I = f(t) \quad \tilde{P}_I = \tilde{\rho}_I \left\{ \frac{3(1 - 8/3\pi r^2 \tilde{\rho}_I)^{\xi_{I/2}} k(t) - 1}{3 - (1 - 8/3\pi r^2 \tilde{\rho}_I)^{\xi_{I/2}} k(t)} \right\}$$

$$P_{\perp}^I - (P_r)_I = \frac{h_I(\tilde{\rho}_I + \tilde{P}_I)(4\pi r^3 \tilde{P}_I + m_I)}{r - 2m_I}$$

**Mantle: Tolman VI--like model**

$$\tilde{\rho}_{II} = \frac{3g(t)}{r^2} \quad \tilde{P}_{II} = \frac{\tilde{\rho}_{II}}{3} \left[ \frac{1 - 9D(t)r\sqrt{4-3\xi_{II}}}{1 - D(t)r\sqrt{4-3\xi_{II}}} \right]$$

$$P_{\perp}^{II} - (P_r)_{II} = \frac{h_{II}(\tilde{\rho}_{II} + \tilde{P}_{II})(4\pi r^3 \tilde{P}_{II} + m_{II})}{r - 2m_{II}}$$

# Surface equations

$$\dot{A} = F\Omega \quad \dot{F} = \frac{2(1-F)F\Omega + 2F(1+\Omega)\hat{E}}{A}$$

$$\dot{\Omega} = TOV(r = A)$$

$$\dot{c}^2 = \frac{(N-1)\alpha F^2}{6\beta} \left\{ \frac{c}{A} \left[ \frac{\beta}{8(F-1)} \right]^{2/\sqrt{4-3\xi_{II}}} \right\}^{\frac{4(1-F)}{3F}}$$

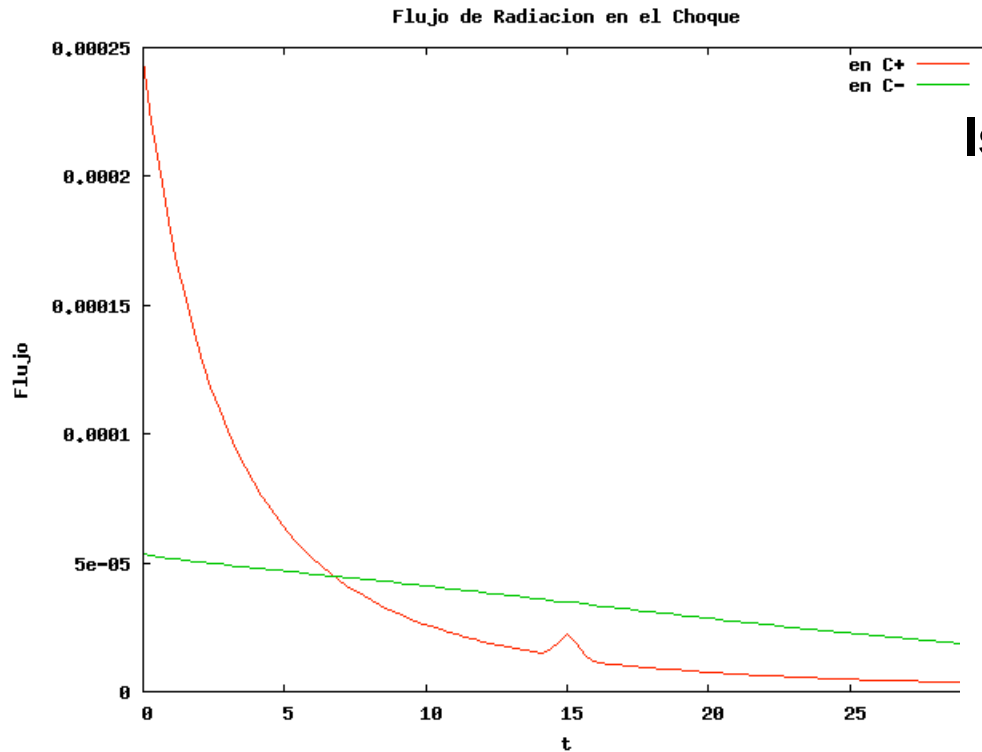
with

$$M = \frac{m_a}{m_a(0)}, \quad A = \frac{a}{m_a(0)}, \quad F = 1 - \frac{2M}{A}, \quad t \rightarrow \frac{t}{m_a(0)}$$

and

$$\hat{E} = 4\pi a^2 \mathcal{F}_a, \quad L = -\dot{M}, \quad E = -\frac{\dot{M}}{F}, \quad \Omega = \omega_a$$

# Anisotropy and radiant shocks



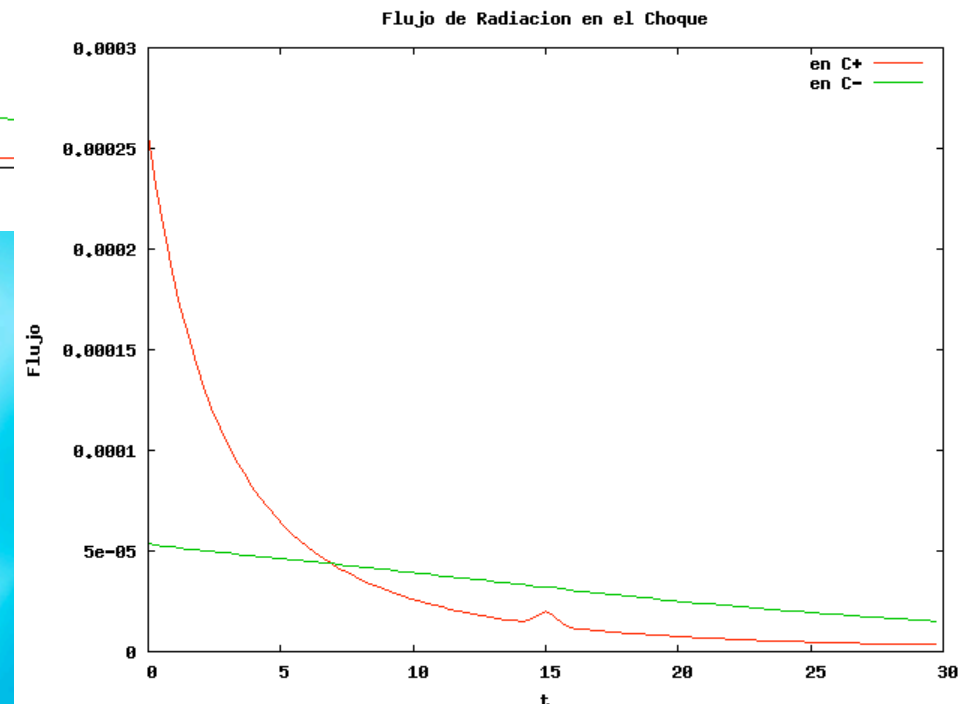
Isotropic nucleus anisotropic mantle  
Diffusive nucleus,  
Free streaming out mantle

$$\Delta = ani_M - ani_N = -0.4$$

$$\Delta = ani_M - ani_N = -0.2$$

$$\frac{M}{a} \approx 0,09372071$$

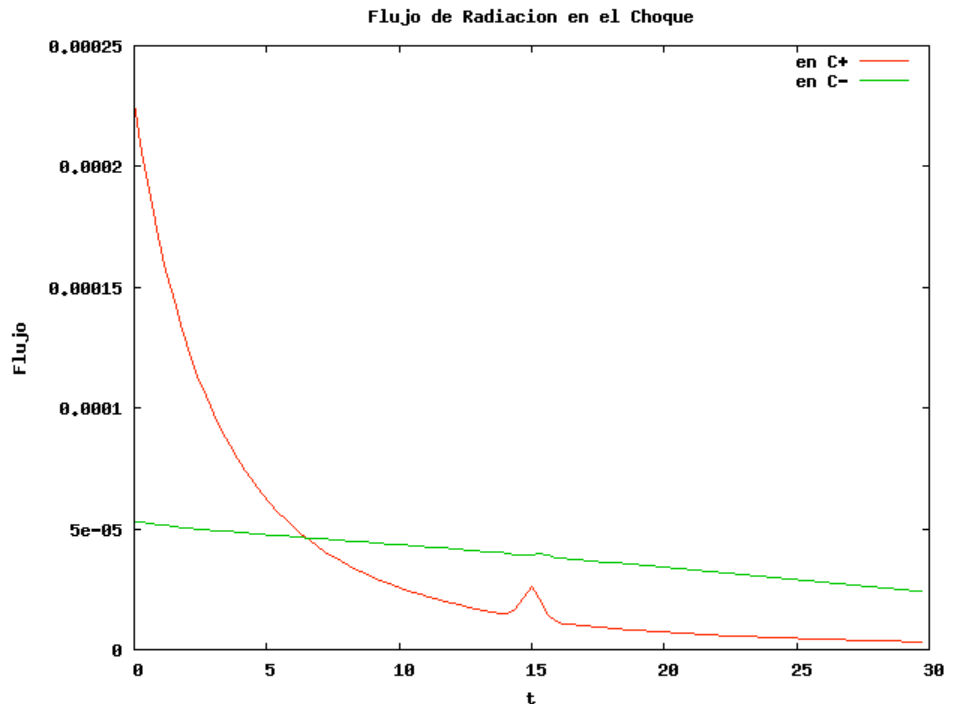
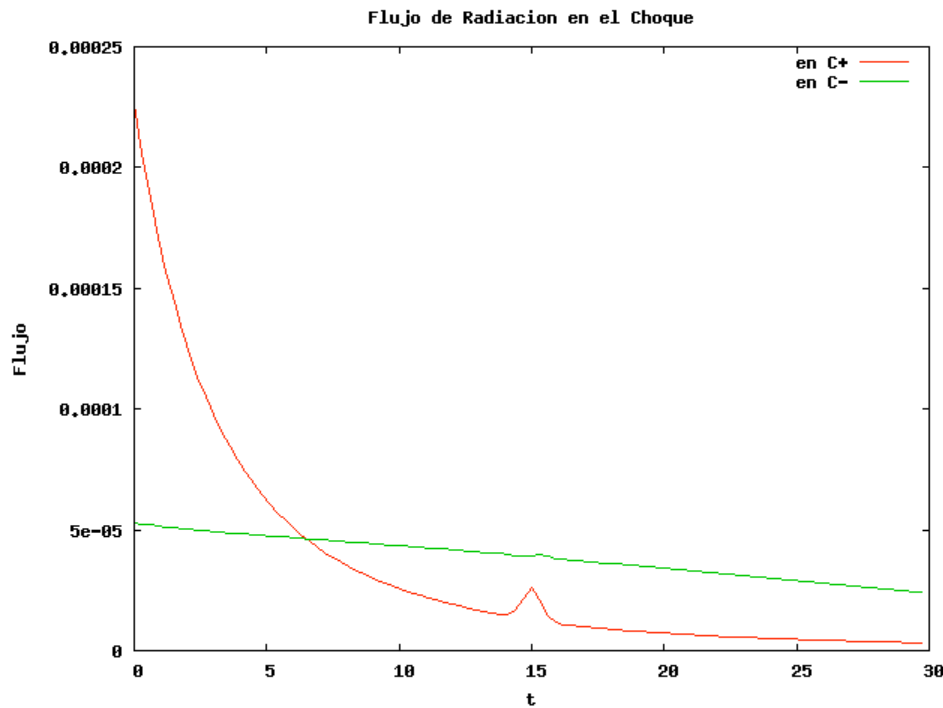
$$\frac{C}{A} \approx 0,24367385$$



# Anisotropy and radiant shocks

anisotropic nucleus isotropic mantle  
 Diffusive nucleus,  
 Free streaming out mantle

$$\Delta = ani_M - ani_N = 0.4$$



$$\Delta = ani_M - ani_N = 0.2$$

$$\frac{M}{a} \approx 0,09372071$$

$$\frac{C}{A} \approx 0,24367385$$

# So far we have found that....

- ⊙ The radiant shock has a “phase transition” during its evolution. First radiates and later it absorbs energy from the mantle before the luminosity peaks. It has to be further explored its relation to the anisotropic jump.
  - ⊙ The peak of luminosity it is recorded very weakly at the nucleus side.
  - ⊙ Anisotropic nucleus seems to induce more opaque configurations, i.e. Nucleus and mantle
- ## Thanks