Cracking of Self-Gravitating Compact Objects with a Non Local Equation of State

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Non Local Equation of state

$$P_r(r) = \rho(r) - \frac{2}{r^3}\int_0^r \bar{r}^2 \rho(\bar{r})\ \mathrm{d}\bar{r}\ + \frac{\mathcal{C}}{2\pi r^3}; \quad \mathcal{C} = \text{const};$$

Which can be re written as $P_r(r) = \rho(r) - \frac{2}{3}\left\langle \rho \right\rangle_r + \frac{\mathcal{C}}{r^3}$

where $\left\langle \rho \right\rangle_r = \frac{\int_0^r 4\pi\bar{r}^2\rho(\bar{r})\ \mathrm{d}\bar{r}}{\frac{4\pi}{3}r^3} = \frac{M(r)}{V(r)}$

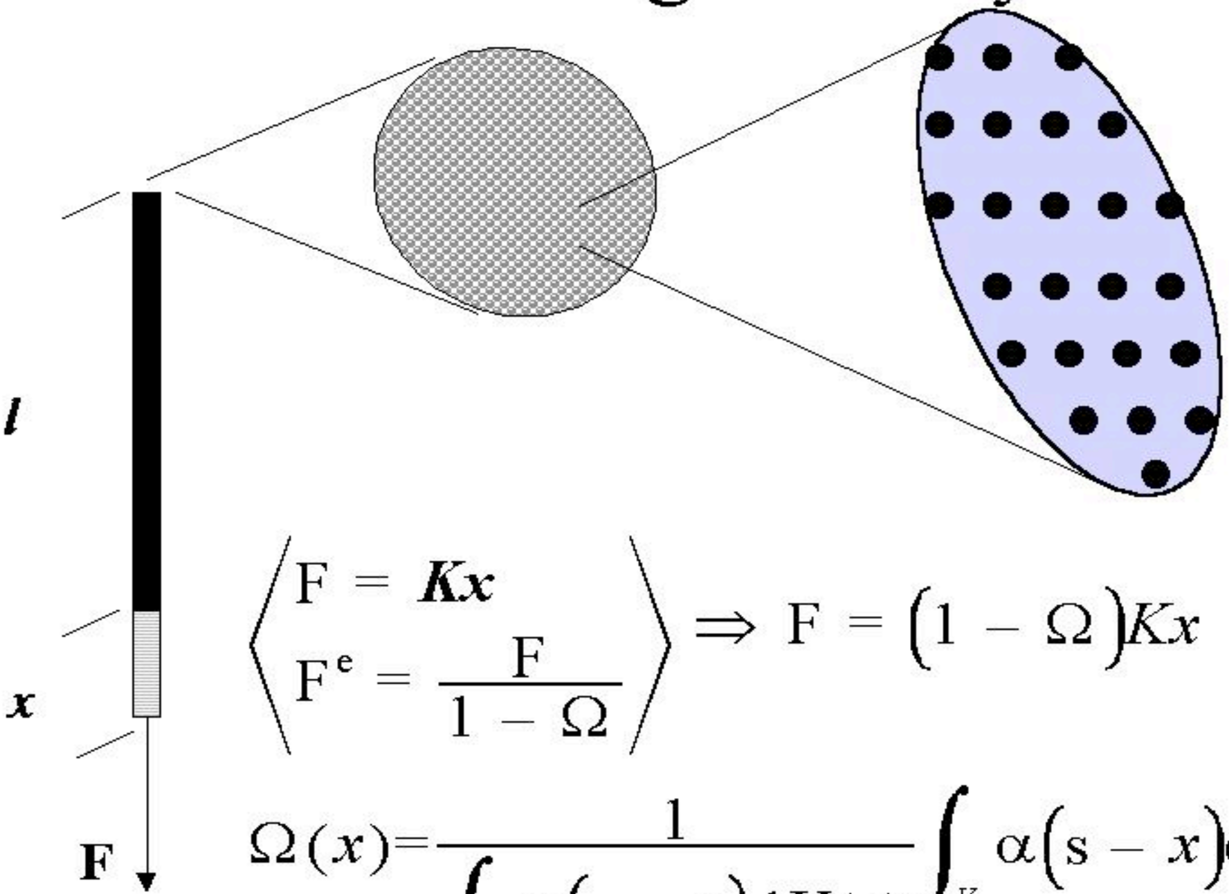
It is a sort of average density, but further more

$$P_r(r) = \mathcal{P}(r) + 2\sigma_{\mathcal{P}(r)} + \frac{\mathcal{C}}{r^3} \quad \text{where} \begin{cases} \mathcal{P}(r) = \frac{1}{3}\rho(r) \quad \text{and} \\ \sigma_{\mathcal{P}(r)} = \left(\frac{1}{3}\rho(r) - \frac{1}{3}\left\langle \rho \right\rangle_r\right) = \left(\mathcal{P}(r) - \bar{\mathcal{P}}(r)\right) \end{cases}.$$

Hernández, Núñez & Percoco CQG, 16, 871 (1999).
Henández & Núñez *Can. J. Phys* 82, 1, (2004).

Material Sciences, Radiation Hydrodynamics, Blood,,,,

No-local Damage Theory



$$\left\langle \begin{array}{l} F = \boldsymbol{Kx} \\ F^{e} = \dfrac{F}{1-\Omega} \end{array} \right\rangle \Rightarrow F = (1-\Omega)Kx$$

$$\Omega(x) = \frac{1}{\int_V \alpha(s-x)\,dV(s)} \int_V \alpha(s-x)\,\omega(s)\,dV(s)$$

The metric and Field Equations

$$\mathrm{d}s^2 = \left(1 - 2\frac{m(r)}{r}\right)^{-1} \left(\mathrm{d}t^2 - dr^2\right) - r^2 \mathrm{d}\Omega^2$$

and the field equations

$$8\pi\rho = \frac{2m'}{r^2}$$

$$8\pi P_r = \frac{2m'}{r^2} - \frac{4\left(m - \mathcal{C}\right)}{r^3}$$

$$8\pi P_\perp = \frac{m''}{r} + \frac{2\left(m'r - m\right)}{r^3}\left[\frac{m'r - m}{r - 2m} - 1\right]$$

Tendencies to crack q-static spherical matter configurations

$$\frac{\mathrm{d}\Theta(r)}{\mathrm{d}r} = R_{\mu\nu}u^{\mu}u^{\nu} + \left(\frac{\mathrm{d}u^{\mu}}{\mathrm{d}r}\right)_{;\mu} + \left(\Omega^{\mu\nu}\Omega_{\mu\nu} - \sigma^{\mu\nu}\sigma_{\mu\nu}\right) + \frac{1}{3}\Theta^{2}$$

Tidal forces immediately after perturbation implies $\frac{\mathrm{d}\Theta}{\mathrm{d}\,r} \neq 0$

And it is equivalent to the sign of the perturbed total force

$$\mathcal{R} = \frac{\mathrm{d}\,P_r}{\mathrm{d}\,r} + (\rho + P_r)\left(\frac{m + 4\pi r^3 P_r}{r\,(r - 2m)}\right) - \frac{2}{r}\,(P_\perp - P_r) \neq 0$$

$$\mathrm{sign}\,[\mathcal{R}] \Leftrightarrow \mathrm{sign}\,[\Theta]$$

Herrera *Phys. Lett A,* **165**, 296 (1992); Di Prisco et al *Phys. Lett A,* **195,** 23 (1994)
Di Prisco, Herrera and Varela *GRG* **29**, 10 (1997)

Tendencies to crack
q-static spherical matter configurations

We shall study the influence of densitiy & anisotropic fluctuations

$$\mathcal{R} = \mathcal{R}\left(\rho_0 + \delta\rho, P_r, \Delta_0 + \delta\Delta\right) = \mathcal{R}_0\left(\rho_0, P_r, \Delta_0\right) + \tilde{\mathcal{R}}\left(\rho_0, P_r, \Delta_0, \delta\rho, \delta\Delta\right)$$

$$P_r(r) = \rho(r) - \frac{2}{r^3}\int_0^r \bar{r}^2 \rho(\bar{r})\, \mathrm{d}\bar{r}$$

Non local

$$P_r(r) = \rho^{\Gamma}(r)$$

Local Politropic

From a known density profile the anisotropic factor is obtained

$$\rho(r) \quad \Rightarrow \left\{ \begin{array}{l} P_r(r) = \rho(r) - \frac{2}{r^3}\int_0^r \bar{r}^2 \rho(\bar{r}) \mathrm{d}\bar{r} \\ P_r(r) = \rho^{\Gamma}(r) \end{array} \right\} \quad \Rightarrow \Delta$$

Density Profiles

Stewart *J Phys. A.* **15**, 2419 (1982)

$$\rho = \frac{1}{8\,\pi r^2}\frac{\left(e^{2Kr}-1\right)\left(e^{4Kr}+8Kre^{2Kr}-1\right)}{\left(e^{2Kr}+1\right)^3}$$

Gokhroo & Mehra GRG, **26**, 75 (1994)

$$\rho = \frac{\sigma}{8\pi}\left[1-K\frac{r^2}{a^2}\right]$$

Wyman *Phys. Rev.* **75**, 1930 (1949)

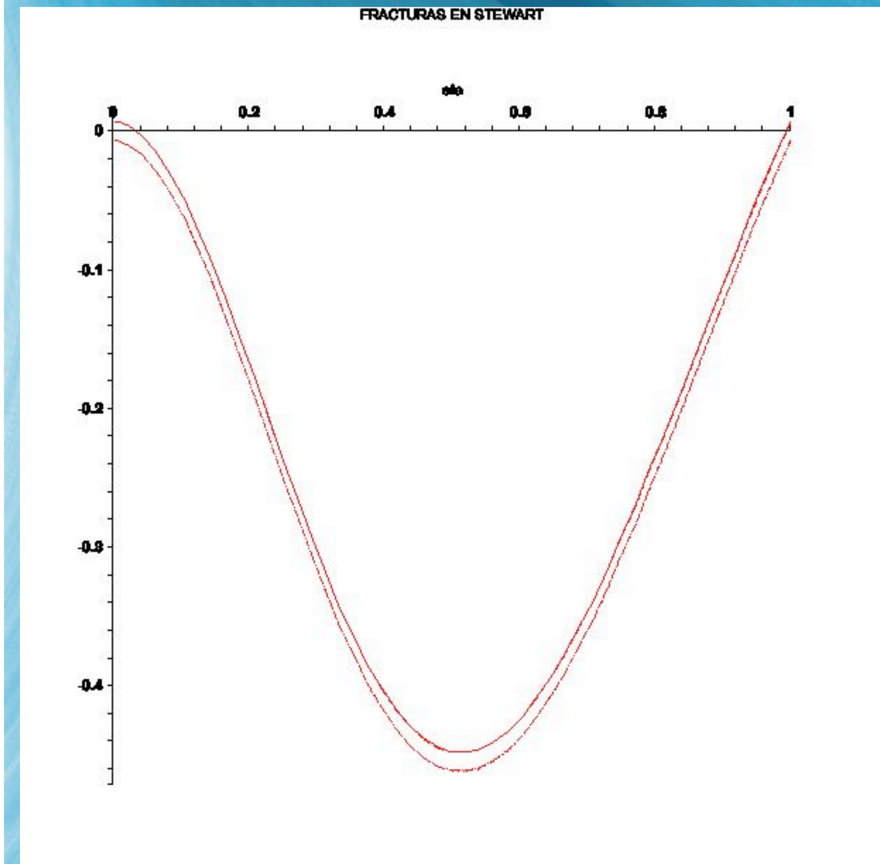
$$\rho = -\frac{C}{8\,\pi}\frac{K\,(3+5\,x)}{(1+3\,x)^{\frac{5}{3}}} \qquad x = C\,r^2$$

Equation of State	M/a	$M\ (M_\odot)$	z_a	$\rho_a \times 10^{14}\ (gr.cm^{-3})$	$\rho_c\ \times 10^{15}\ (gr.cm^{-3})$
Example 1	0.32	2.15	0.6	6.80	1.91
Example 2	0.40	2.80	1.2	8.84	1.99
Example 3	0.38	2.54	1.0	8.04	2.41

Delgaty + Lake (1998), *Comput. Phys. Commun.*, **115**, 395
http://xxx.lanl.gov/abs/gr-qc/9809013.

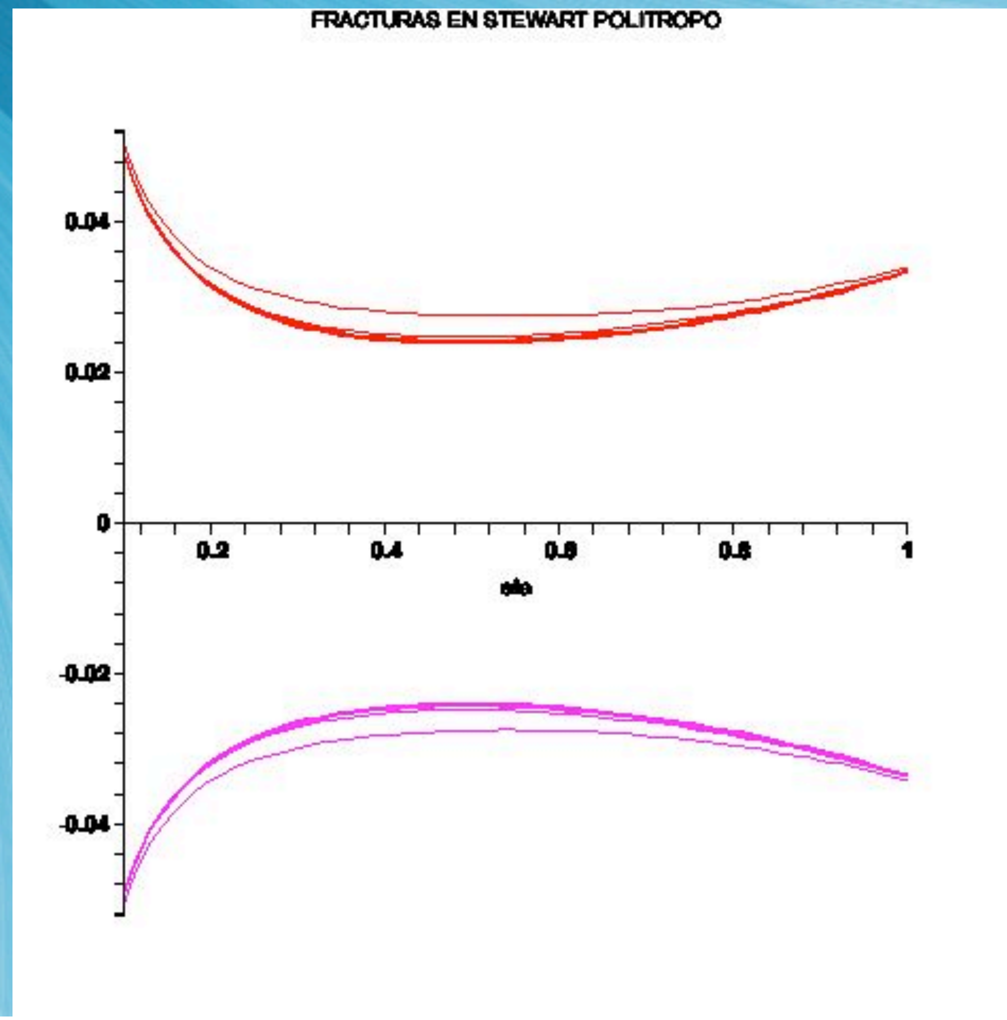
Stewart *J Phys. A.* **15**, 2419 (1982)

$$\rho = -\frac{C}{8\pi}\,\frac{K\,(3+5\,x)}{(1+3\,x)^{\frac{5}{3}}} \qquad x = C\,r^2$$

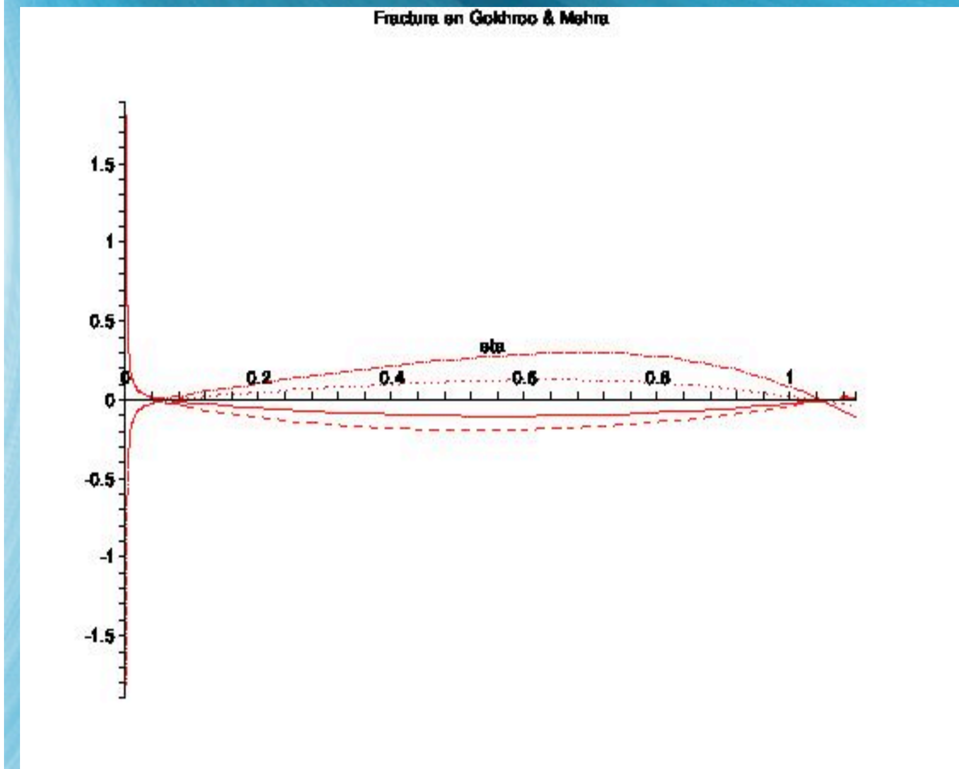


Non local

Local Politropic

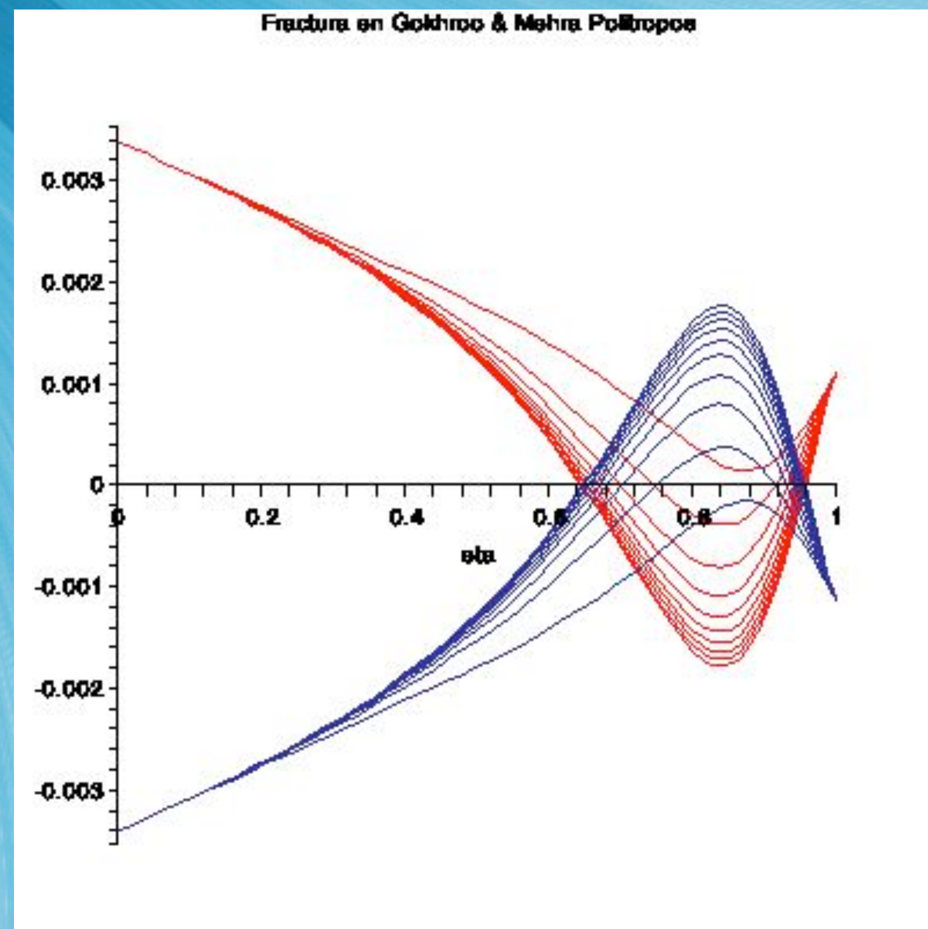


Cracking Gokhroo & Mehra GRG, **26**, 75 (1994))



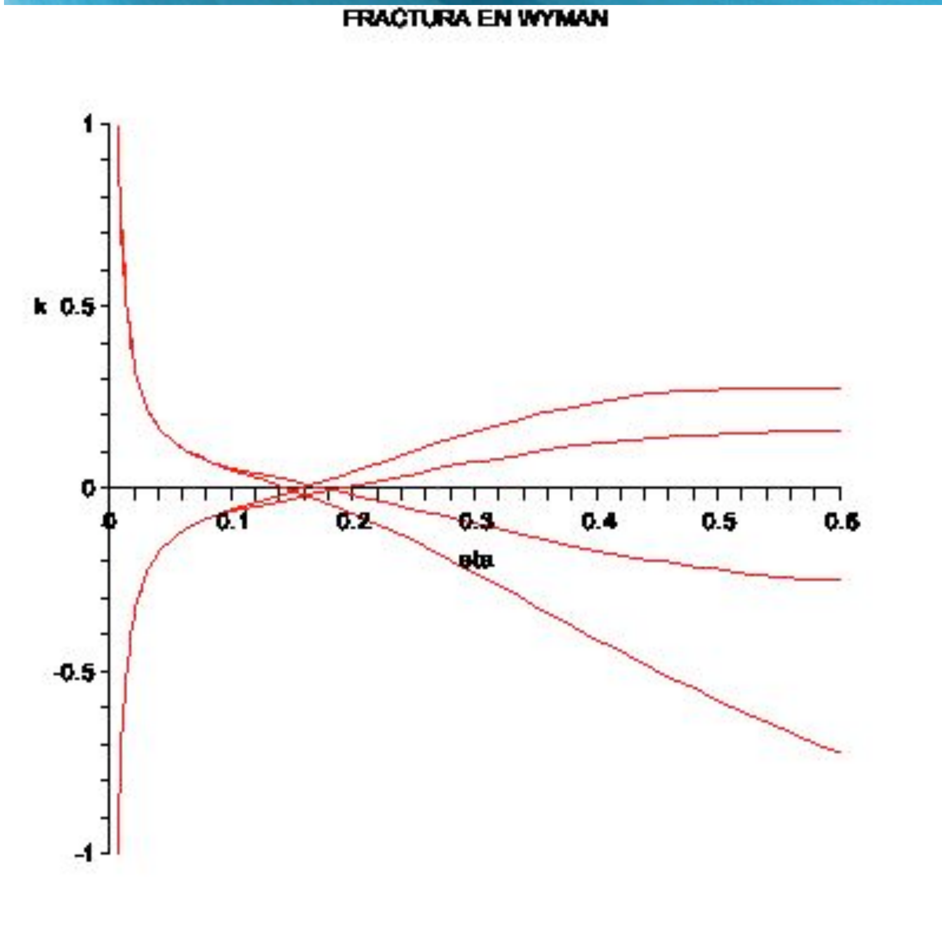
$$\rho = \frac{\sigma}{8\pi}\left[1 - K\frac{r^2}{a^2}\right]$$

Local Politropic



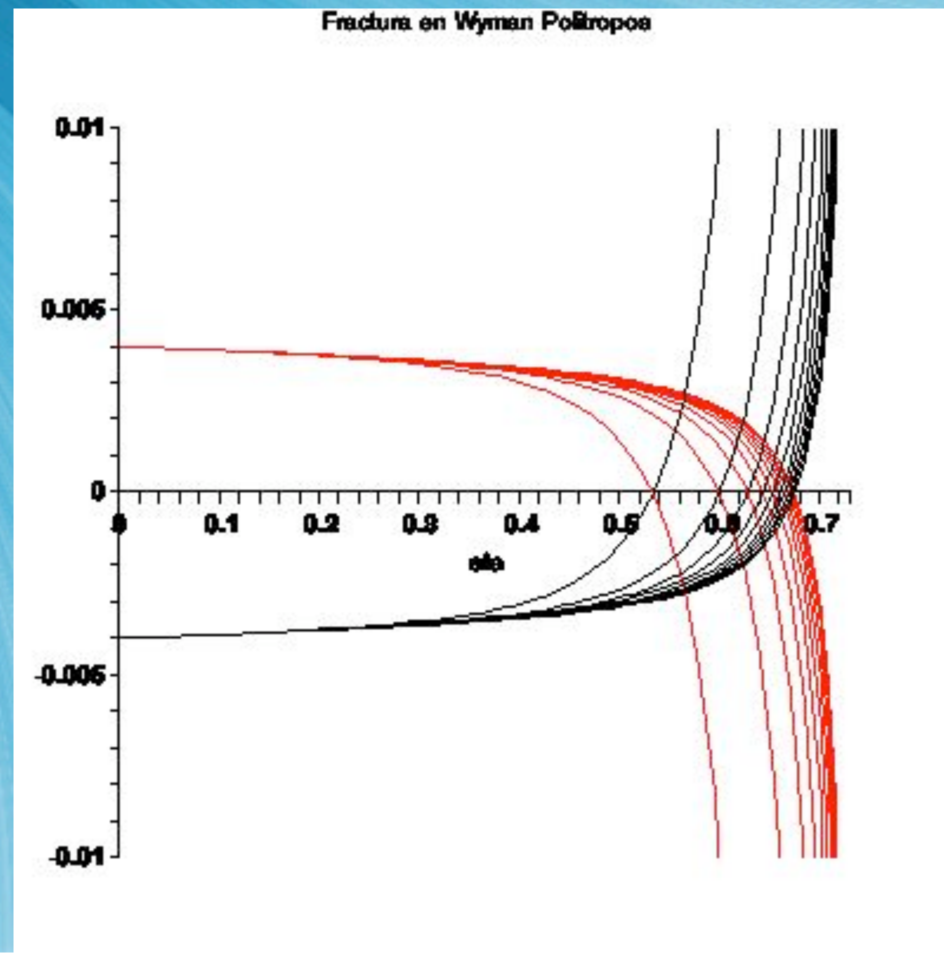
Non local

Cracking Wyman *Phys. Rev.* **75**, 1930 (1949)



$$\rho = -\frac{C}{8\pi}\frac{K(3+5x)}{(1+3x)^{\frac{5}{3}}} \qquad x = C\,r^2$$

Local Politropic



Non local