Guía de Ejercicios # 20

1.- Resuelva la ecuación diferencial respectiva empleando el método de separación de variables.

(1) $\dfrac{dy}{dx} = \operatorname{sen}(5x)$

(2) $\dfrac{dy}{dx} = (x+1)^2$

(3) $dx + e^{3x}dy = 0$

(4) $dx - x^2 dy = 0$

(5) $(x+1)\dfrac{dy}{dx} = x + 6$

(6) $e^x \dfrac{dy}{dx} = 2x$

(7) $xy' = 4y$

(8) $\dfrac{dy}{dx} + 2xy = 0$

(9) $\dfrac{dy}{dx} = \dfrac{y^3}{x^2}$

(10) $\dfrac{dy}{dx} = \dfrac{y+1}{x}$

(11) $\dfrac{dy}{dx} = \dfrac{x^2y^2}{1+x}$

(12) $\dfrac{dy}{dx} = \dfrac{1+2y^2}{y\operatorname{sen}(x)}$

(13) $\dfrac{dy}{dx} = e^{3x+2y}$

(14) $\dfrac{dy}{dx} = \dfrac{x}{y^2\sqrt{1-x}}$

(15) $\dfrac{dy}{dx} = 3x^2(1+y^2)^{3/2}$

(16) $e^x y\dfrac{dy}{dx} = e^{-y}e^{-2x-y}$

(17) $2y(x+1)dy = xdx$

(18) $x^2y^2dy = (y+1)dx$

(19) $y\ln(x)\dfrac{dx}{dy} = \left(\dfrac{y+1}{x}\right)^2$

(20) $(1 + x^2 + y^2 + x^2y^2)dy = y^2dx$

(21) $\dfrac{dy}{dx} = \left(\dfrac{2y+3}{4x+5}\right)^2$

(22) $\dfrac{dS}{dr} = kS$

(23) $\dfrac{dQ}{dt} = k(Q-70)$

(24) $\dfrac{dP}{dt} = P - P^2$

(25) $\dfrac{dN}{dt} + N = Nte^{t+2}$

(26) $\sec^2(x)dy + \csc(y)dx = 0$

(27) $\operatorname{sen}(3x)dx + 2y\cos^3(3x)dy = 0$

(28) $\sec(x)dy = x\cot(y)dx$

(29) $\dfrac{y}{x}\dfrac{dy}{dx} = (1+x^2)^{-1/2}(1+y^2)^{1/2}$

(30) $(y - yx^2)\dfrac{dy}{dx} = (y+1)^2$

(31) $\dfrac{dy}{dx}\dfrac{1}{y} = \dfrac{2x}{y}$

(32) $\dfrac{dy}{dx} = \dfrac{xy + 3x - y - 3}{xy - 2x + 4y - 8}$

(33) $\dfrac{dy}{dx} = \dfrac{xy + 2y - x - 2}{xy - 3y + x - 3}$

(34) $x\sqrt{1-y^2}dx = dy$

(35) $y(4-x^2)^{1/2}dy = (4+x^2)^{1/2}dx$

(36) $(4y + yx^2)dy - (2x + xy^2)dx = 0$

(37) $e^y\operatorname{sen}(2x)dx + \cos(x)(e^{2y} - y)dy = 0$

(38) $(e^y+1)^2e^{-y}dx + (e^x+1)^3e^{-x}dy = 0$

(39) $\dfrac{dy}{dx} = \operatorname{sen}(x)(\cos(2y) - \cos^2(y))$

(40) $\sec(y)\dfrac{dy}{dx} + \operatorname{sen}(x-y) = \operatorname{sen}(x+y)$

2.- Use el método de separación de variables, para resolver la ecuación diferencial dada, sujeta a la condición inicial respectiva.

(1) $\dfrac{dy}{dx} = 5x^4 - 3x^2 - 2$, $y(1) = 4$

(2) $\dfrac{dy}{dx} = xy^3(1+x^2)^{-1/2}$, $y(0) = 1$

(3) $\dfrac{dy}{dt} + ty = y$, $y(1) = 3$

(4) $ydy = 4x(y^2+1)^{1/2}dx$, $y(0) = 1$

(5) $\dfrac{dx}{dy} = 4(x^2+1)$, $x\left(\dfrac{\pi}{4}\right) = 1$

(6) $\dfrac{dy}{dx} = \dfrac{y^2-1}{x^2-1}$, $y(2) = 2$

(7) $\dfrac{dy}{dx} = \dfrac{x^2-1}{y^2+1}$, $y(-1) = 1$

(8) $\dfrac{dy}{dx} = \dfrac{x^2+1}{2-y}$, $y(-3) = 4$

(9) $\dfrac{dy}{dx} = \dfrac{x + xy^2}{4y}$, $y(1) = 0$

(10) $\dfrac{dy}{dx} = -\dfrac{3x + xy^2}{2y + x^2y}$, $y(2) = 1$

(11) $y' = x + 3$; $x(0) = 2$.

(12) $y' = 2^{x+y}$; $y(0) = 1$.

(13) $x' = \dfrac{e^t}{(1+e^t)x}$; $x(0) = 1.$

(14) $x^2y' = y - xy$; $y(-1) = -1$

(15) $y' + 2y = 1$; $y(0) = \frac{5}{2}$

(16) $y' \operatorname{sen}(x) = y\ln(y)$; $y(\frac{\pi}{2}) = e$

(17) $\cos(x)dx + (1 + e^{-x})\operatorname{sen}(y)dy = 0$; $y(0) = \frac{\pi}{4}$

(18) $3e^x\tan(y)dx + (2 - e^x)\sec^2(y)dy = 0$; $y(0) = \frac{\pi}{4}$

(19) $(e^{-y} + 1)\operatorname{sen}(x)dx = (1 + \cos(x))dy$, $y(0) = 0$

(20) $(1 + x^4)dy + x(1 + 4y^2)dx = 0$, $y(1) = 0$

3.- Determine la solución de la ecuación diferencial $\dfrac{dy}{dx} - y^2 = -9$, y luego hallar en cada caso, una solución particular que pase por:

(a) $(0, 0)$ (b) $(0, 3)$ (c) $(\frac{1}{3}, 1)$

4.- Resuelva cada una de las ecuaciones diferenciales homogéneas.

(a) $(x - y)dx + xdy = 0$

(b) $(x + y)dx + xdy = 0$

(c) $xdx + (y - 2x)dy = 0$

(d) $(y^2 + yx)dx - x^2dy = 0$

(e) $(y^2 + yx)dx + x^2dy = 0$

(f) $-ydx + (x + \sqrt{xy})dy = 0$

(g) $(xy + y^2)dx - x^2dy = 0$

(h) $x\dfrac{dy}{dx} - y = \sqrt{x^2 + y^2}$

(i) $ydx = 2(x + y)dy$

(j) $\dfrac{dy}{dx} = \dfrac{y - x}{y + x}$

(k) $\dfrac{dy}{dx} = \dfrac{x + 3y}{3x + y}$

(l) $\dfrac{dy}{dx} = \dfrac{4y - 3x}{2x - y}$

(m) $\dfrac{dy}{dx} = \dfrac{x - 3y}{x - y}$

(n) $\dfrac{dy}{dx} = -\dfrac{4x + 3y}{2x + y}$

(ñ) $\dfrac{dy}{dx} = \dfrac{x^2 - y^2}{3xy}$

(o) $\dfrac{dy}{dx} = \dfrac{x^2 + xy + y^2}{x^2}$

(p) $\dfrac{dy}{dx} = \dfrac{x^2 - 3y^2}{2xy}$

(q) $\dfrac{dy}{dx} = \dfrac{x^2 + 3y^2}{2xy}$

(r) $\dfrac{dy}{dx} = \dfrac{3y^2 - x^2}{2xy}$

(s) $\dfrac{dy}{d\theta} = \dfrac{\theta\sec\left(\frac{y}{\theta}\right) + y}{\theta}$

21. $(3x^2 - y^2)dx + (xy - x^3y^{-1})dy = 0$

5.- Resuelva la ecuación homogénea, sujeta a la condición inicial respectiva.

(a) $\dfrac{dy}{dx} = \dfrac{3x + 2y}{x}$; $y(1) = 2$

(b) $\dfrac{dy}{dx} = \dfrac{x^3 + y^3}{xy^2}$; $y(1) = 1$

(c) $xy^2\dfrac{dy}{dx} = y^3 - x^3$; $y(1) = 2$

(d) $\dfrac{dy}{dx} = \left(\dfrac{x}{y} + \dfrac{y}{x}\right)$; $y(1) = -4$

(e) $(x^2 + 2y^2)\dfrac{dx}{dy} = xy$; $y(-1) = 1$

(f) $(x + ye^{y/x})dx - xe^{y/x}dy = 0$; $y(1) = 0$

(g) $(2y^2 + 4x^2)dx - xydy = 0$; $y(1) = -2$

(h) $ydx + x(\ln(x) - \ln(y) - 1)dy = 0$; $y(1) = e$